

100

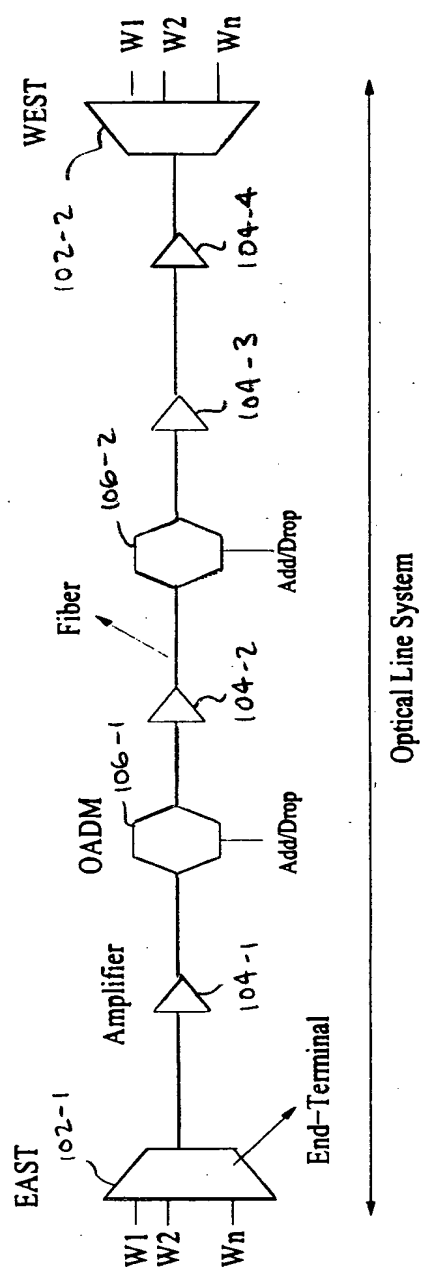


FIG 1

Special Case	
problem	Complexity
$(L, *, E), s = 2$ $ C_2 = \infty$	polynomial
$(L, *, NE), s = 2$	polynomial
$(L, U, E), s = 2$	4/3-Approx
$(L, D, *), s = 3$	NP-hard

General Case		
problem	Approx lower bound	Approx upper bound
$(L, D, *)$	$\Omega(\sqrt{s})$	$O(\sqrt{s})$
(L, U, NE)	$1 + 1/s^2$	2
(L, U, E)	NP-hard	2
$(C, *, NE)$	in-approximable	
(C, D, E)	in-approximable	
(C, U, E)	NP-hard	$2(1 + \epsilon)$

(A)

(B)

FIG. 2

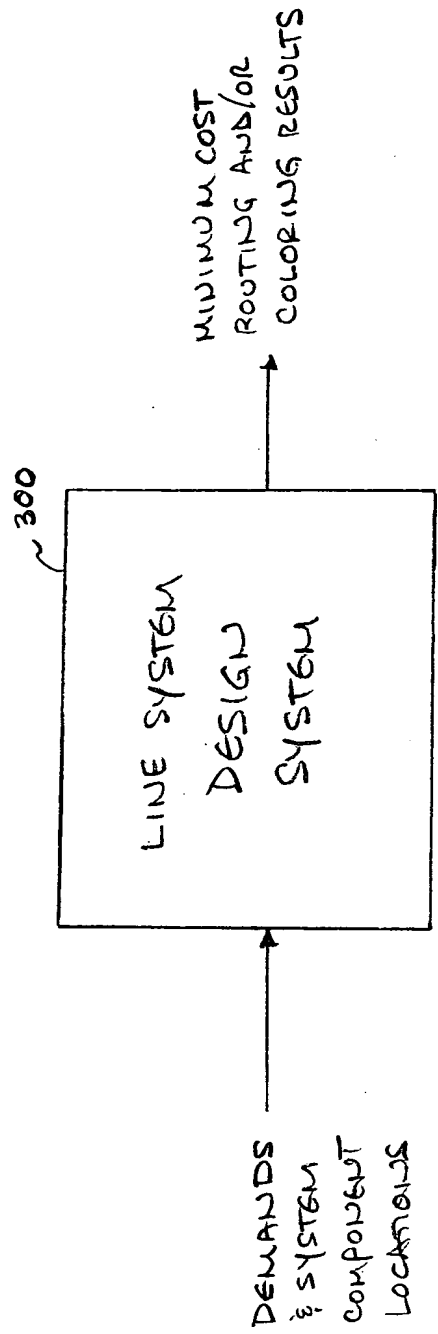


FIG. 3

Methodology A $m(0) = 0$ for $p = 1$ to line system load/2

{

 $l(p) = 0; m(p) = m(p - 1) + 2;$ for $i = 1$ to $n - 1$

{

 $l_i(p) \leftarrow$ load on link e_i if ($l_i(p) = 0$)

{

Divide the line system into two line systems;
 one from node 0 to node ($i - 1$); the other from
 node i to node ($n - 1$) and call methodology A
 on these line systems separately.

}

if ($l_i(p) > l(p)$)

{

 $l(p) = l_i(p)$

}

}

create a multigraph $G = (V, E)$, where $V = \{0, \dots, n - 1\}$ for all demand (i, j) in D

{

create an edge $(i - 1, j)$ in G

}

for $i = 1$ to $n - 1$

{

if $l_i(p) < l(p)$ add an edge $(i - 1, i)$ in G

}

set the capacity of each edge in G to 1find a 2-unit flow from node 0 to node ($n - 1$) in G Let p_1 and p_2 be the path for the flowFor all the demands corresponding to links in p_1 .

{

Assign the color $c_{m(p)}$ to demandremove the demand from D

}

For all the demands corresponding to links in p_2

{

Assign the color $c_{m(p)+1}$ to demandremove the demand from D

}

}

FIG. 4A

Routing Phase:

if $(L(R_s) \geq n(1 + \epsilon)/\epsilon)$

Output R_s

else {

Compute $D_1 = \{d \in D \mid d \text{ in any routing goes through at least } n/3 \text{ links} \}$

Compute $D_2 = D - D_1$

Compute $R_1 =$ the set of all possible routings for demands in D_1

Compute $R_2 =$ the set of all possible routings for demands in D_2

in which at most $3S$ demands are not routed on shortest paths

Compute $R = R_1 \times R_2$

Compute $r \in R$ such that $L(r) = \min_{r' \in R} L(r')$

Output r

}

Coloring Phase:

$U = D$

$M =$ the set of available colors

$l = \min_{e_i \in L} l_i(U)$ (the min. load of demands in U)

while $(l > 0)$ {

Compute $O = H(U)$ (see below)

Compute $m = \{i, j \mid i, j \text{ are the smallest two colors in } M \}$

Color demands in O with colors in m

$U = U - O$

$M = M - m$

$l = \min_{e_i \in L} l_i(U)$

}

if $(U \neq \emptyset)$ {

Color U using methodology A

"Compute $O = H(U)$ ":

Compute $d_0 =$ a demand in U that goes through the largest number of links in L

$O = \{d_0\}$

$L' =$ set of links covered by demands in O

$i = 1$

while $(L' \neq L)$ {

Compute $D_i = \{d \mid d \in U - O \text{ \& } d \text{ overlaps with } d_{i-1}\}$

Compute $d_i = \{d \mid d \in D_i \text{ \& } d \text{ goes through the largest number of links in } L - L'\}$

$i = i + 1$

output O

}

FIG. 4B

Methodology B:

```

 $e_0 = (-1, 0)$ 
 $e_{n+1} = (n, n+1)$ 
 $L = L \cup \{e_0, e_{n+1}\}$ 
 $D = D \cup \{(0, 0), (n+1, n+1)\}$ 
for all  $(0 \leq i \leq j \leq n+1)$  {
   $P(i, j) = \emptyset$ 
   $R(i, j) = \emptyset$ 
  best = 0
  for all  $(i \leq i' \leq j' \leq j)$  {
     $E_1 = \{e_i, e_{i+1}, \dots, e_{i'}\} \cup \{e_{j'+1}, e_{j'+2}, \dots, e_j\}$ 
     $E_2 = \{e_{i'+1}, e_{i'+2}, \dots, e_{j'}\}$ 
    Compute coloring  $C$  using methodology b1 where  $E_1$  ( $E_2$ ) links are colored with 1 (2)
    steps
    if( $C \neq \emptyset$ ) {
      if( $i' - i + j - j' + 1 \geq \text{best}$ ) {
         $R(i, j) = C$ 
        best =  $i' - i + j - j' + 1$ 
      }
    }
  }
}
}
Compute  $L_1 = \{e_i \mid e_i \in L, l_i \leq |C_1|\}$ 
for all  $(e_i, e_j \in L_1)$  {
  Compute  $D_{i,j} = \{d \mid d \in D, d \text{ goes through either link } e_i, e_j\}$ 
  Compute  $P_{i,j} = \text{coloring obtained by coloring the interval graph } D_{i,j} \text{ with colors in } C_1$ 
}
for all  $(e_i, e_j \in L_1, i < j)$  {
  best = 0
  for all  $(m, i < m < j)$  {
    Compute the coloring  $K = P(i, m) + P(m, j)$ 
    If( $K = \emptyset$ ) continue
    Compute  $n = \text{number of links that are in one step in } K$ 
    if(best <  $n$ ) {
      best =  $n$ 
       $C = K$ 
    }
  }
}
Compute  $n = \text{number of links that are in one step in } R(i, j)$ 
if(best <  $n$ ) {
  best =  $n$ 
   $C = R(i, j)$ 
}
}
 $P(i, j) = C$ 
}
Output  $P(0, n+1)$ 

```

FIG. 4C

Methodology b1:

Compute C = interval graph coloring of demands D_1 using colors in C_1

if($C == \emptyset$) **Output** C

Compute C' = interval graph coloring of the demands in $D - D_1$ using first available colors

Output $C' \cup C$

FIG. 4D

Methodology c1:

$V = \{0, 1, \dots, n-1\}$

$E = \emptyset$

for all demands $((i, j) \in D - D_1)$ {

$E = E \cup \{(i-1, j)\}$

Directed link $(i-1, j)$ has unit capacity

}

for all links $(e_i \in L)$ {

$E = E \cup \{(i-1, i)\}$

Directed link $(i-1, i)$ has capacity $|C_1| + |C_2| - l_i$

}

Graph $G = (V, E)$

Compute maxFlow = Max. Flow f in G from node 0 to node $n-1$

if(maxFlow < $|C_2|$) **Output** \emptyset

Compute $F_1 = \{d \mid f \text{ puts zero flow on the edge } (i-1, j) \text{ where demand } d = (i, j)\}$

Compute $F_1 = F_1 \cup D_1$

Compute K_1 = coloring that colors demands in F_1 with colors in C_1 only using interval graph coloring

Compute K_2 = coloring that colors demands in $D - F_1$ with colors in C_2 only using interval graph coloring

Output $K = K_1 \cup K_2$

FIG 4E

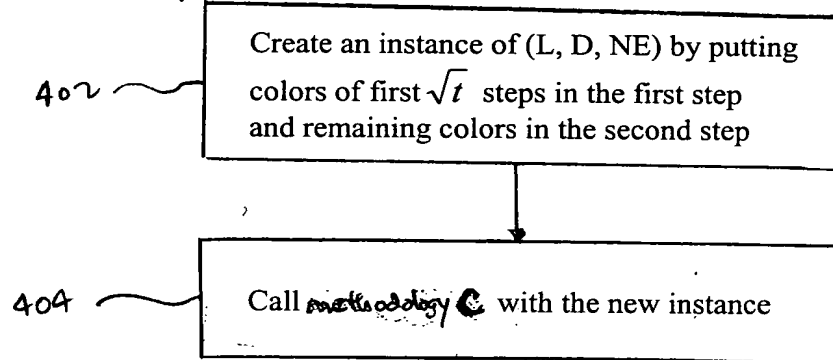
Methodology D:

FIG. 4F

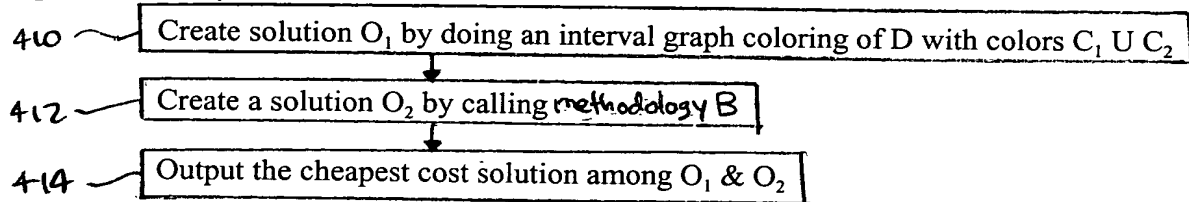
Methodology E:

FIG. 4G

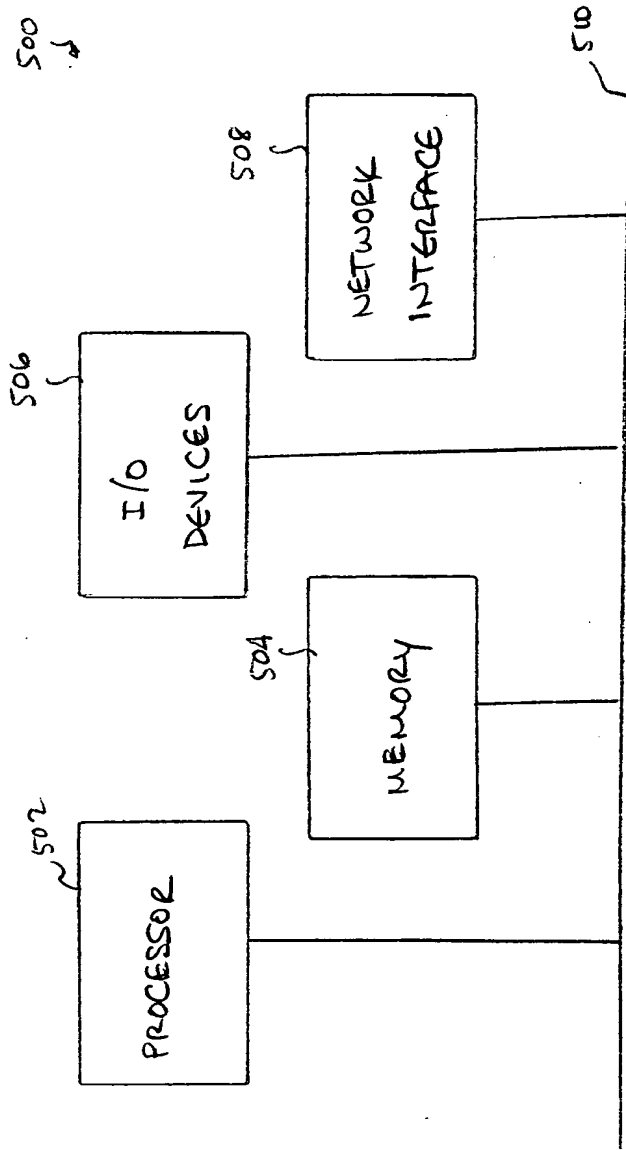


FIG. 5